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# Economic Growth and Unemployment —Theoretical Foundations of Okun's Law—

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## Abstract

There is a widely accepted empirical regularity, called Okun's law, that predicts negative relationship between changes in the unemployment rate and changes in output growth. However, growth theory usually assumes full employment, since it focuses on long-run phenomena. There is a need for medium run macroeconomic theory that focuses on persistent unemployment or stagnation with relation to economic growth. For this purpose, this paper constructs a model of economic growth that includes the unemployment rate as an endogenous variable. The dynamic equation of this model is reduced to the relation between changes in the unemployment rate and changes in output growth, which gives theoretical foundations of Okun's law. This theoretical relation is tested by using the data of the U.S. and Japan.

## 1 Introduction

The modern macroeconomics is usually divided into two subarea: the long-run theory and the short-run theory. The former analyzes economic growth and the latter business cycles. The long-run growth theory usually deals with full-employment economy, leaving the problems of unemployment to the short-run business cycle theory. The standard growth model, represented by the Solow model,<sup>1)</sup> predicts that the rate of growth is independent of the rate of unemployment, since it includes no unemployment. However, there is an well-known empirical law called Okun's law that predicts a negative relationship between the rate of change in the unemployment rate and the rate of change in output. In his original paper published in 1962,<sup>2)</sup> Okun found that every 1 percentage point reduction was associated with additional output growth of 3 percentage point in the US economy. The exact quantitative form of this relationship is different between countries, and it also changes over time. As a matter of fact, the current version of Okun's law in the US is stated that a 1 percentage point decline in the unemployment rate is associated with additional output growth of about 2 percent. However, the negative correlation between

changes in unemployment rate and changes in output growth is viewed as one of the most consistent relationship in macroeconomics. If this is the case, it is desirable to have a growth model that consistently explains this negative relationship.

The experiences of OECD countries since the beginning of 1980's has revealed that unemployment is not only a short-run phenomenon but also a medium-run phenomenon. European countries has suffered from a high rate of unemployment around 10% more than 25 years since the beginning of 1980's. Japan also suffered from a high unemployment rate around 5% more than 10 years since the middle of 1990's compared to a low rate around 3% before then. It is also pointed out in many literature that changes in income distribution in recent years are not easily analyzed by business cycle models or growth models. These facts indicate that modern economies are characterized by medium-run evolutions that are quite distinct from either short-run business cycles or steady-state growth. From this view point, Blanchard (1997) calls macroeconomic changes that spread over periods of 15 to 30 years as medium-run phenomena, and suggests the importance of developing macroeconomics of the medium-run.<sup>3)</sup> Solow (2000) also mentions the need to develop the medium-run macroeconomic theory that explains medium-run departure from the steady growth.<sup>4)</sup> For this purpose, he suggests the idea of using Okun's law in growth theory, saying "what is wanted is an integration of Okun's law and growth models, so that the events of the business cycle are directly linked to the evolutions of the growth path".<sup>5)</sup> This is not only useful for growth theory, but also for Okun's law, because "Okun's law might be improved by this marriage, too".<sup>6)</sup>

In this paper, we construct a medium-run growth model that integrate Blanchard's idea about labor market into the Solow model. In contrast to the Solow model, our model provides an appropriate framework for studying the determination of the rate of unemployment in the growth process. This model also provides a theoretical foundation of Okun's law, i.e., the negative relationship between changes in the rate of unemployment and the rate of output growth. So far, Okun's law remains to be an empirical observation rather than a result derived from theory.<sup>7)</sup> Moreover, this quantitative relationship varies depending on the countries and time periods under consideration. To identify what factors cause these differences, the theory that explains this empirical law is required.

The paper is organized as follows. In the next section, we construct a model of economic growth in which the unemployment rate is endogenously determined. The model may be characterized as an extension of the Solow model, because it includes the latter as its extreme case. Section 3 examines the dynamics of growth and employment in the medium run using this model. Section 4 derives Okun's law as a theoretical relationship from this model. Section 5 tests this theoretical relationship using the data of the U.S. and Japan. It is shown that the theoretical relationship is helpful to explain the substantial difference in the Okun's coefficients between the U.S. and Japan.

## 2 A Model of Economic Growth Including Unemployment

In this section, we construct a medium-run macroeconomic model that explains both growth and unemployment. The model developed below basically adopts the framework of the Solow model, but modifies it so as to include unemployment. The production function with labor augmenting technological progress is given by

$$Y = F(AN, K) \quad ((1))$$

where  $Y$  is output,  $N$  is labor employment,  $K$  is capital stock and  $A$  is the efficiency of labor. Assuming that the production function is subject to the constant returns to scale, it is rewritten as

$$y = f(n) \quad ((2))$$

where  $y$  is output per unit of capital and  $n$  is the efficiency labor per unit of capital, i.e.

$$y \equiv \frac{Y}{K}, \quad n \equiv \frac{AN}{K}. \quad ((3))$$

The function  $f(n)$  is assume to have the ordinary well-behaved properties.

As with the Solow model, we assume that a constant proportion of income is saved and invested in capital, so that the growth rate of capital is given by

$$\frac{\dot{K}}{K} = sf(n) - \delta K \quad ((4))$$

where  $s$  is the proportion of income saved and  $\delta$  is the rate of depreciation of capital. The labor augmenting technological progress is assumed to be proceeding at a constant rate,  $\alpha$ :

$$\frac{\dot{A}}{A} = \alpha. \quad ((5))$$

The structure of the model formulated so far is the same as that of the Solow model. However, the treatment of the labor market in our medium-run macroeconomic model is different from that of the Solow model. The Solow model assumes that wages and prices are always market-clearing, so that the economy always achieves full employment. On the contrary, the medium-run macroeconomic model uses the price-setting and the wage-setting equations to explain the determination of employment and the real wage rate. Therefore, it may be more reasonable to assume that the economy composed of monopolistically competitive price-setting firms rather than perfectly competitive price-taking firms. For explanatory convenience, we assume that each firm uses one unit of capital, which is combined with variable amounts of labor to produce output. Then, the production function of a firm is given by (2). The capital stock is equal to the number of firms in the economy, and consequently changes in the

capital-stock due to (4) correspond to the entry or exit of firms. It should be noted that  $n$  is both employment in a given firm and the ratio of labor to capital for the economy as a whole.

As each firm is monopolistically competitive in the goods market, it faces the downward sloping demand curve. The demand for its good is assumed to be given, in the inverse form, by

$$p = \left( \frac{y}{\bar{y}} \right)^{-\eta}; 0 \leq \eta < 1, \quad ((6))$$

where  $p$  is the price charged by the firm relative to the price level,  $\bar{y}$  is average output of all firms, and  $\eta$  is the inverse of the elasticity of demand. At each point in time, a firm determines the amount of labor  $n$  to maximize profit  $\pi$  defined as:

$$\pi = py - \left( \frac{w}{A} \right) n, \quad ((7))$$

where  $w$  is the real wage rate in terms of the price level. The first order condition and the symmetry condition that all firms must charge the same price, so that  $p = 1$ , imply that

$$\left( \frac{1}{\mu} \right) f'(n) = \frac{w}{A} \quad ((8))$$

where  $\mu = 1/(1-\eta)$  is the markup of price over marginal cost. For any given real wage rate, this equation determines the demand for labor of each firm. Then the aggregate demand for labor is given by  $N = (n/A)K$ , the ratio of labor to capital times the number of firms.

Let us next consider the supply side of the labor market. The Solow model assumes the supply of labor to grow at a constant rate independently of the wage rate. In the medium-run macroeconomic model, by contrast, the supply side of labor market is represented by a wage-setting equation, with wages tending to exceed the market-clearing level. The wage-setting equation is derived from efficiency wage or bargaining models. These theoretical models of wage-setting generate a strong core implication that the tighter the labor market, the higher the real wage, given the workers' reservation wage. The simplest formulation of the wage-setting is given by the following equation which was proposed by Blanchard (1997):<sup>8)</sup>

$$\frac{w}{A} = \beta \left( \frac{N}{N_S} \right)^\epsilon \quad ((9))$$

where  $w$  is the real wage rate,  $A$  is the efficiency coefficient,  $N$  is labor employment,  $N_S$  is labor population, and  $\beta$  is the parameter that reflect reservation wages or the bargaining power of laborers. This equation implies that the real wage rate increases with the rate of employment which reflects the tightness of the labor market. The parameter  $\epsilon$  represents the sensitivity of the real wage rate to the tightness of the labor market.

Denote the ratio of labor population in efficiency unit  $AN_s$  to capital  $K$  by  $n_s$ :

$$n_s \equiv \frac{AN_s}{K}. \quad ((10))$$

Then, equation (9) is rewritten, by using this and the definition of  $n$  in (3), as:

$$\frac{w}{A} = \beta \left( \frac{n}{n_s} \right)^\epsilon. \quad ((11))$$

From (8) and (11), which are the demand for and the supply of labor equations, the equilibrium of the labor market is given as follows:

$$\left( \frac{1}{\mu} \right) f'(n) = \beta \left( \frac{n}{n_s} \right)^\epsilon \quad ((12))$$

At a given point in time,  $n_s$  is constant since the population of labor in efficiency unit  $AN_s$  and capital stock  $K$  are given. Thus, this equation determines  $n$ , which must be less than  $n_s$  in efficiency wage or bargaining models. Then labor employment is determined by  $N = (n/A)K$ .

Labor population  $N_s$  is assumed to grow at constant rate  $\lambda$ , i.e.,

$$\frac{\dot{N}_s}{N_s} = \lambda. \quad ((13))$$

From (4), (5) and (13), the rate of change of  $n_s$  over time is given by

$$\frac{\dot{n}_s}{n_s} = (\alpha + \lambda + \delta) - sf(n) \quad ((14))$$

Now, the system consisting of equations (12) and (14) determines  $n$  and  $n_s$  over time. Then the time path of the employment rate  $n/n_s$ , which we will denote by  $z$  hereafter, is also determined.

The dynamic equation (14) appears to be similar to that of the Solow model. Actually, it is different from the latter, because equation (14) includes two variables  $n$  and  $n_s$  while in the Solow model  $n$  and  $n_s$  are always the same. As is mentioned above, the employment-capital ratio  $n$  that is determined by equation (12) must be less than  $n_s$  under the efficiency and bargaining hypotheses. When  $\epsilon \rightarrow \infty$  in (12), which means the real wage rate is completely flexible, we must have  $n = n_s$ . Thus, the continuous equilibrium in the labor market is ensured when  $\epsilon \rightarrow \infty$ . In this case, equation (14) becomes as

$$\frac{\dot{n}}{n} = (\alpha + \lambda + \delta) - sf(n) \quad ((15))$$

which is the same as the dynamic equation of the Solow model. In addition, if we assume  $\mu = 1$  ( $\eta \rightarrow 0$ ) in (8) and (12), the model reduces to the Solow model. In this sense, our model includes the Solow model as an extreme case.

### 3 The Dynamics of Growth and Employment in the Medium Run

Since we are concerned with analyzing the dynamics of the growth rate and the employment rate in the medium run, it will be more convenient to rewrite the model consisting of (12) and (14) in terms of the employment rate  $z$ , which is defined as:

$$z \equiv \frac{n}{n_s}. \quad ((16))$$

Using this notation, equation (12) is rewritten as:

$$\left(\frac{1}{\mu}\right) f'(n) = \beta z^\epsilon \quad ((17))$$

Differentiating (16) with respect to time, we obtain the following equation:

$$\frac{\dot{z}}{z} = \frac{\dot{n}}{n} - \frac{\dot{n}_s}{n_s}. \quad ((18))$$

Similarly, taking logarithm of the both sides of equation (17) and differentiating it with respect to time, we get

$$\frac{\dot{n}}{n} = -\frac{\epsilon \sigma(n)}{1 - \theta(n)} \frac{\dot{z}}{z}. \quad ((19))$$

where  $\sigma(n)$  is the elasticity of substitution between labor and capital and  $\theta(n)$  is the elasticity of output with respect to employment, which are defined as follows:

$$\sigma(n) \equiv -\frac{f'(n)[f(n) - nf'(n)]}{nf''(n)f(n)}, \quad \theta(n) \equiv \frac{nf'(n)}{f(n)} \quad ((20))$$

It follows from the ordinary assumptions about the production function  $f(x)$  that

$$\sigma(n) > 0, \quad 0 < \theta(n) < 1 \quad ((21))$$

Substituting (14) and (19) into (18), and solving it with respect to  $\dot{z}/z$ , we obtain the following equation:

$$\frac{\dot{z}}{z} = \frac{1 - \theta(n)}{1 - \theta(n) + \epsilon \sigma(n)} [sf(n) - (\alpha + \lambda + \delta)] \quad ((22))$$

The ratio of employment to capital  $n$ , which appears on the right-hand side of this equation, can be expressed as a function of  $z$ , the employment rate, by solving equation (17):

$$n = n(z), \quad ((23))$$

the derivative of which becomes as:

$$n'(z) = -\frac{\epsilon \sigma(n)z}{\{1 - \theta(n)\}n} < 0. \quad ((24))$$

Substituting (23) into (22), we have the following equation for the evolution of  $z$ :

$$\frac{\dot{z}}{z} = \phi(z)[sf(n(z)) - (\alpha + \lambda + \delta)] \quad ((25))$$

where

$$\phi(z) = \frac{1 - \theta(n(z))}{1 - \theta(n(z)) + \varepsilon\sigma(n(z))} > 0. \quad ((26))$$

The dynamics of this model can be analyzed from equation (25).

In view of (21), the following condition holds for any relevant value of  $z$ . Thus the steady state value of  $z^*$  must satisfy the following condition:

$$sf(n(z^*)) - \delta = \alpha + \lambda, \quad ((27))$$

which means that the rate of growth of capital at  $z^*$  (the left-hand side) is equal to the rate of growth of labor force in efficiency unit (the right-hand side). It is easy to check from this condition that the rate of employment in the steady state  $z^*$  is higher, the higher the rate of saving  $s$ , the lower the rate of technological progress  $\alpha$ , and the lower the growth rate of population  $\lambda$ . In addition, parameters  $\beta$ ,  $\varepsilon$  and  $\mu$  in equation (17) also affect the steady state value of  $z^*$  through their influences on the functional form of  $n(z)$ . It is shown that  $z^*$  is higher, the lower the bargaining power of the labor union  $\beta$ , the lower the mark-up of price over marginal cost  $\mu$ , and the higher the elasticity of the real wage rate to the employment rate (i.e., the flexibility of the real wage rate)  $\varepsilon$ . The steady state condition (27) is the same as that of the Solow model except that  $n$  is expressed as a function of  $z$  through (17). This relation gives new insights concerning the influences of changes in the parameters of the model on the rate of employment that cannot be analyzed by the Solow model.

This steady state equilibrium can be shown to be stable. Note that  $df(n(z))/dz = f'(n)n'(z) < 0$ . From this inequality and (26), we have

$$\frac{d}{dt} \left( \frac{\dot{z}}{z} \right)_{z=z^*} = \phi(z^*)sf'(n(z^*))n'(z^*) < 0, \quad ((28))$$

which implies that the steady state is locally stable. It may be intuitively clearer to see this result by a graphical representation. In figure 1, the downward sloping curve represents the growth rate of capital stock as a function of the employment rate, while the horizontal line at  $\alpha + \lambda$  represents the growth rate of labor force in efficiency unit. The value of  $z$  at their intersection,  $z^*$ , is the employment rate at the steady state. When  $z < z^*$ , the growth rate of capital stock exceeds the growth rate of the labor force, so that  $z$  increases towards  $z^*$ . When  $z > z^*$ , in contrast, the growth rate of capital falls below the growth rate of labor force in efficiency unit, so that  $z$  decreases towards  $z^*$ . Thus the system approaches towards the steady state, starting from any initial condition.

## 4 Theoretical Foundations of Okun's Law

Equation (25) can be used to give theoretical foundation of Okun's law. As is mentioned before, Okun's law is an empirical law which states that there is a



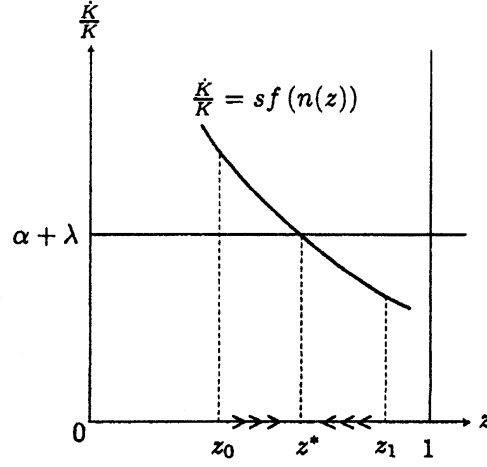


Figure 1: Dynamics of the Employment Rate

negative relationship between changes in the unemployment rate and changes in output growth. As far as I know, however, there is no literature that gives theoretical explanations of this law. I will show below that this relationship is derived theoretically from our model.

Taking into account the relation  $sf(n(z)) - \delta = \dot{K}/K$ , equation (25) can be rewritten as

$$\frac{\dot{z}}{z} = \phi(z) \left[ \frac{\dot{K}}{K} - (\alpha + \lambda) \right]. \quad ((29))$$

This equation indicates that there is a positive relationship between the rate of growth of capital and the rate of changes in the employment rate, which is reduced to a negative relationship between the rate of growth of capital and the rate of unemployment  $u$ , since  $u = 1 - z$ . In order to relate this equation to Okun's law, the growth rate of capital stock  $\dot{K}/K$  must be replaced by the growth rate of output  $\dot{Y}/Y$ .

In view of (16), the production function (2) may be written as

$$\frac{Y}{K} = f(zn_s). \quad ((30))$$

Taking logarithmic differentiation of this function with respect to time, we have

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} + \theta(n) \left( \frac{\dot{z}}{z} + \frac{\dot{n}_s}{n_s} \right), \quad ((31))$$

where  $\theta(n)$  is as defined by (20). Using (18), (19) to rewrite (31), we have

$$\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} + \frac{\varepsilon \sigma(n) \theta(n)}{1 - \theta(n)} \frac{\dot{z}}{z}. \quad ((32))$$

Substitute this relation into (29), and solve it with respect to  $\dot{z}/z$ , taking into

account (26). Then we have

$$\frac{\dot{z}}{z} = \frac{1}{1 + \varepsilon\sigma(n)} \left[ \frac{\dot{Y}}{Y} - (\alpha + \lambda) \right]. \quad ((32))$$

Finally, replace  $z$  by  $u$ , taking into account  $z = 1 - u$ . Then we get the theoretical equation of Okun's law as follows:

$$\frac{\dot{Y}}{Y} = (\alpha + \lambda) - \frac{1}{1 - u} [1 + \varepsilon\sigma(n)] \dot{u}. \quad ((33))$$

As this equation shows, the rate of growth of output in the case of  $\dot{u} = 0$  is equal to  $\alpha + \lambda$ , which is the steady growth rate of the model. The coefficient for  $\dot{u}$ , which is called Okun's coefficient, is equal to  $[1 + \varepsilon\sigma(n)]/(1 - u)$ . Since this value depends on  $u$ , Okun's coefficient in theory is not constant. However, this value will presumably change little within the relevant range of  $u$ . Thus, it may be reasonable to evaluate the coefficient at the steady state. Then equation (33) is rewritten as

$$\frac{\dot{Y}}{Y} = (\alpha + \lambda) - \frac{1 + \varepsilon\sigma^*}{1 - u^*} \dot{u} \quad ((34))$$

where  $u^*$  and  $\sigma^*$  are given, by using  $z^*$  in equation (27), as follows:

$$u^* = 1 - z^*, \quad \sigma^* = \sigma(n(z^*)) \quad ((35))$$

Now, if you draw the diagram of equation (34), taking  $\dot{u}$  on the horizontal axis and  $\dot{Y}/Y$  on the vertical axis, it is expressed as a downward sloping line with intercept  $\alpha + \lambda$  on the vertical axis and with slope  $-(1 + \varepsilon\sigma^*)/(1 - u^*)$ . Therefore, if the rate of unemployment remains the same, real GDP grows by  $\alpha + \lambda$ , which is the normal growth in the production of goods and services resulting from technological progress and growth in the labor force. In addition, for every percentage point the unemployment rises, real GDP growth falls by  $(1 + \varepsilon\sigma^*)/(1 - u^*)$  percent. This result shows that the value of Okun's coefficient depends on the elasticity of the real wage rate to the unemployment rate  $\varepsilon$  and the elasticity of substitution between labor and capital  $\sigma^*$ , given the rate of unemployment at the steady state  $u^*$ ; this coefficient is larger, the larger the product of the two parameters  $\varepsilon\sigma^*$ .

## 5 Testing the Theory<sup>9)</sup>

In his original research, Okun found that a 1 percentage point decline in the unemployment rate was, on average, associated with additional output growth of about 3 percentage points for the U.S. economy. In recent years, Okun's law is widely accepted as stating that a 1 percentage point decrease in the unemployment rate is associated with additional output growth of 2 percent, as is illustrated in the text books of macroeconomics like Mankiw's. There are changes in the value of Okun's coefficient not only over time but also between

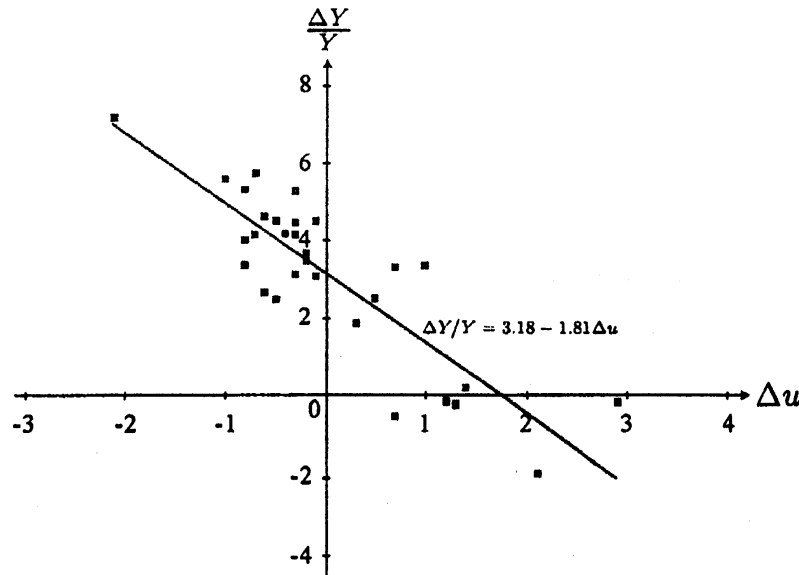


Figure 2: Okun's Law in the U.S. Economy

countries. Figure 2 and 3 each represent scatterplot of the change in the unemployment rate on the horizontal axis and the percentage change in real GDP on the vertical axis, using annual data from 1969 to 2000 on the U.S. economy and the Japanese economy, respectively. Each figure shows clearly that there is a negative correlation between these variables.

To be more precise about the magnitude of Okun's law relationship, we have estimated the Okun's equation for U.S. and Japan to obtain the following results:

$$\text{U.S.: } \frac{\Delta Y}{Y} = \frac{3.18}{(16.6)} - \frac{1.81}{(9.19)} \Delta u, \quad (R^2 = 0.738) \quad ((36))$$

$$\text{Japan: } \frac{\Delta Y}{Y} = \frac{3.61}{(9.18)} - \frac{6.18}{(4.27)} \Delta u + \frac{5.13}{(5.74)} DUM, \quad (R^2 = 0.657), \quad ((37))$$

where  $t$ -statistics is reported in the parenthesis below each estimated parameter. Estimated equation (36) for the U.S. economy shows more or less the same result as stated above. In equation (37) for Japan, we added the dummy variable ( $DUM$ ) that takes 1 for the years from 1969 to 1973 and zero otherwise to distinguish those years as included in the high growth era. As a matter of fact, the year of 1973 is regarded as the end of the high growth era in Japan, as the real GDP growth declined sharply after that.

Comparing the results of regressions for the U.S. and Japan, we notice that there is a significant difference in the magnitude of Okun's coefficients between the two countries. In the U.S., a 1 percentage point decrease in the unemployment rate is associated with additional output growth of 1.81 percent, while, in Japan, it associated with additional output growth of 5.13 percent. Why is there so much difference in this parameter between the two countries? If we

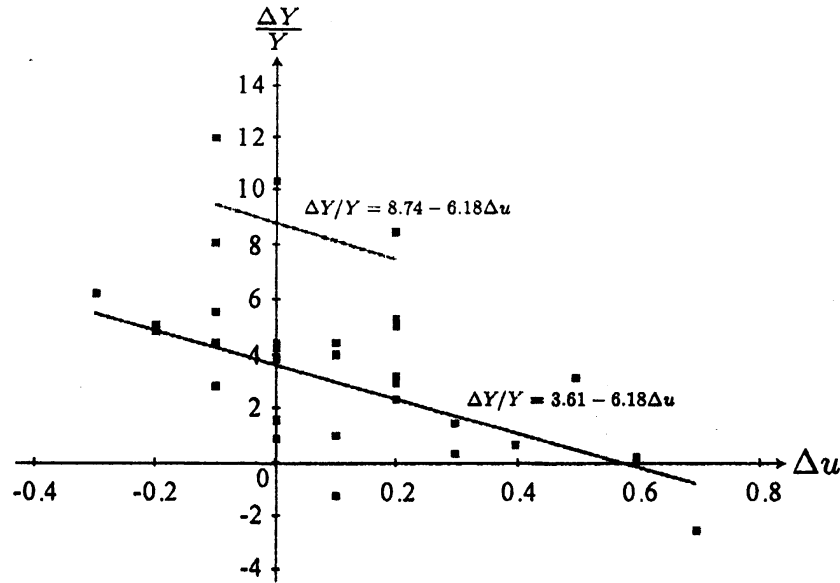


Figure 3: Okun's Law in the Japanese Economy

apply the theoretical equation (34) to the estimated results, we must have

$$\frac{1 + \varepsilon\sigma^*}{1 - u^*} = 1.81 \text{ for the U.S.}, \quad \frac{1 + \varepsilon\sigma^*}{1 - u^*} = 5.13 \text{ for Japan} \quad ((38))$$

Suppose, for simplicity, that the unemployment rate at the steady state  $u^*$  is equal to 0.04 in both countries. It follows that  $\varepsilon\sigma^*$  must be equal to 0.73 for the U.S. and 3.92 for Japan. This implies that the values of  $\varepsilon$  or  $\sigma^*$  (or both) must be higher in Japan than in the U.S.. In other words, the reason why a 1 percentage-point decrease in the unemployment rate is associated with higher additional output growth in Japan than in the U.S. is that the elasticity of the real wage to the unemployment rate or the elasticity of substitution between labor and capital (or both) are higher in Japan than in the U.S..

Whether this holds true or not may be checked by estimating the values of  $\varepsilon$  and  $\sigma$  for the U.S. and Japan. As for  $\sigma$ , most of the econometric studies for the U.S. and Japan shows that it is less than unity. So we may assume that  $0 < \sigma^* \leq 1$  holds for both countries. This together with the values of  $\varepsilon\sigma^*$  obtained above implies  $\varepsilon \geq 0.73$  for the U.S. and  $\varepsilon \geq 3.92$  for Japan. To examine the values of  $\varepsilon$ , we need to make regression analyses for the following equation which is derived by taking logarithm of equation (9):

$$\log w = at + \varepsilon \log(1 - u). \quad ((39))$$

where  $at$  indicates the time trend. We estimated this equation by simple OLS, taking sample periods 1964-2005 for the U.S. and 1970-2000 for Japan, to obtain the following results:

$$\text{U.S.: } \log w = -0.00083t + 0.39 \log(1 - u), \quad ((40))$$

$$\text{Japan: } \log w = 0.0084t + 3.38 \log(1 - u). \quad ((41))$$

These results imply that  $\varepsilon = 0.39$  for the U.S. and  $\varepsilon = 3.38$  for Japan. These estimated values are a little lower than those obtained above as the theoretical prediction. However, the result that the value of  $\varepsilon$  is much higher in Japan than in the U.S. is consistent with the prediction of the theory.

## 6 Conclusion

There are many persistent disequilibrium phenomena in the macro-economy, such as persistent unemployment or stagnation, which are not analyzed properly either by the short-run Keynesian model nor by the long-run growth model. In this paper, we constructed a medium run model that might be suitable to analyze these persistent disequilibrium phenomena. The model assumes the goods market to be monopolistically competitive, and the real wage to be sticky in the labor market based on the efficiency wage or bargaining hypotheses. These market imperfections generate unemployment even at the steady state equilibrium. By analyzing the model, we have identified the factors that make the steady state unemployment rate higher. In addition we have shown that the dynamic equation of our model is reduced to an equation that expresses the rate of output growth as a function of changes in the unemployment rate. This equation gives the theoretical foundations for Okun's law that is widely accepted as a robust empirical law. It is shown that the value of Okun's coefficient is explained by the elasticity of the real wage to the unemployment rate (i.e., the real wage flexibility) and the elasticity of substitution between labor and capital. Using the data from the U.S. and Japan, we have tested whether these parameters are appropriate to explain the size of Okun's coefficient. It is shown that the substantial difference of Okun's coefficient between the two countries may be attributed at least partly to the difference in the elasticity of the real wage rate to the unemployment rate, i.e., the real wage flexibility.

However, these two parameters are not enough to explain fully the size of Okun's coefficient. I consider it important to introduce the utilization of labor and capital into the model to achieve more perfect marriage of Okun's law with growth theory. I plan to discuss about this attempt in another paper.

## 7 Notes

- 1) See Solow (1956).
- 2) See Okun (1962).
- 3) See Blanchard (1997) and Blanchard (2000).
- 4) See Solow (2000a) and Solow (2000b).
- 5) See Solow (2000a), p.185.
- 6) See Solow (2000a), p.185.
- 7) Prachowny (1993) attempts to provide theoretical foundations of Okun's law by deriving the relationship between unemployment changes and output from a production function for the economy and ancillary relationships in the labor

market. Quite differently from his approach, this paper attempts to provide theoretical foundations of Okun's law by integrating it with a growth theory.

8) See Blanchard (1997), p.108.

9) I owe the econometric analyses of this section to Professor Ichiro Tokutsu (Konan University).

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